Comparison of fractal models using NMR and CT analysis in low permeability sandstones

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**A B S T R A C T**

Fractal geometry provides an effective method for characterization of the complex and irregular pore structure of Eocene Shahejie low permeability sandstones in the Raoyang Sag, the Bohai Bay Basin, China. Laboratory measurements including porosity, permeability, scanning electron microscope (SEM), thin sections, nuclear magnetic resonance measurements (NMR) and X-ray computed tomography (CT) technology are used to provide insights into the fractal characteristics of pore structure in sandstones of Eocene Shahejie Formation in the Bohai Bay Basin, China. Quantitative CT analysis reveals the pore radius is not always linear to \( T_2 \) (transverse relaxation time) value obtained from the NMR tests, but instead power function of \( T_2 \). Fractal analysis was performed on the \( T_2 \) distribution using various fractal models, and the related fractal dimensions are calculated. The fractal dimensions calculated using various fractal models are correlated with NMR parameters and permeability. The fractal curves break into two segments at the \( T_{2\text{cutoff}} \) (\( T_2 \) separating the immovable and movable fluids) value or smaller when using fractal model I and fractal model II, and only the large-scale pore networks can be described by the fractal geometry. Mostly the entire pore size distributions (micro-pores to large-scale pore networks) can be described by the fractal model III, and the calculated fractal dimensions are in accordance with the CT scanning and thin section data, and are strongly correlated with the \( T_{2\text{gm}} \) (geometric mean of \( T_2 \)), permeability and BVI (bulk volume of immovable fluids). The fractal behaviors of pore size distributions from NMR analysis have implications for pore structure evaluation in low permeability sandstones with similar geological settings.

1. Introduction

Fractal theory has been widely used in pore structure characterization of low permeability sandstones since its initial proposal by Mandelbrot in 1970s (Mandelbrot, 1977; Katz and Thompson, 1985; Hansen and Skjelholt, 1988; Daigle et al., 2014; Lai and Wang, 2015; Lai et al., 2016; Shao et al., 2017; Wang et al., 2018a). The self-similar fractal objects can be quantitatively described by the fractal dimensions (Cai et al., 2010; Anovitz et al., 2013; Lai and Wang, 2015). Fractal dimensions, which represent the irregularity and complexity of the microscopic pore structures (Li and Horne, 2006; Wang et al., 2012; Cai et al., 2017; Yan et al., 2018), can be derived from thin sections and SEM analysis (Katz and Thompson, 1985), capillary pressure curve (Li and Horne, 2003; Li, 2016; Sakaee-Pour and Li, 2016), nitrogen adsorption desorption (Yang et al., 2014; Lai et al., 2018a; Liu et al., 2019), NMR analysis (Daigle et al., 2014; Zhang and Weller, 2014; Shao et al., 2017; Lai et al., 2018b), as well as CT imaging techniques (Li et al., 2016). The fractal dimension of the 3-D pore structures in reservoir rocks can vary from 2.0 (smooth surface) to 3.0 (highly complex surface) (Giri et al., 2012; Yang et al., 2014; Lai and Wang, 2015).

The high-resolution X-ray Computed Tomography (CT) imaging technique could reconstruct the three-dimensional (3D) distribution of pore throat systems and fractures in a nondestructive way (Christe et al., 2011; Bera et al., 2011; Cnudde and Boone, 2013; Nabawy and David, 2016; Lai et al., 2017; Li et al., 2017), and provide morphological information about the pore network such as pore sizes, shapes,
pore connectivity, and tortuosity (Zhao et al., 2018). Laboratory NMR measurements provide uncalibrated but almost the full ranges of the continuous pore size distributions of reservoir rocks (Sigal, 2015; Yan et al., 2017; Lai et al., 2018a; Xiao et al., 2018), and they can be used to quantitatively evaluate the complexity of pore structures with calibration by capillary pressure or CT analysis (Sigal, 2015; Daigle and Johnson, 2016; Zhao et al., 2017; Nabawy et al., 2018; Zhang et al., 2018; Gao et al., 2019). Therefore, the full-ranges of pore size distribution can be characterized when fractal analysis is performed on the NMR \( T_2 \) spectrum.

The major goal of this study is to investigate the fractal characteristics of low permeability sandstones (intrinsic permeability of 0.1–10 mD) of the third member of the Eocene Shahejie Formation (Sha-3 or Es3) in the Raoyang Sag, the Bohai Bay Basin, Eastern China through various fractal models performed on NMR measurements. The intrinsic permeability of 0.1–10 mD corresponds to the poor to fair permeability in classification of Abuamarah et al. (2019). The pore structure characteristics (geometry, pore throat size distribution, pore connectivity) of the sandstones are investigated using a combination of thin section, SEM, NMR measurement and CT scanning data. The pore size distributions (\( T_2 \) distributions) from NMR analysis, which are represented by fractal scaling, are used to calculate the fractal dimensions through various fractal models. The fractal behaviors of the NMR \( T_2 \) spectrum are linked with the pore-networks. Regression analysis was performed to reveal the relationships between pore structure parameters and fractal dimensions calculated by various fractal models. The work helps improve the understanding of fractal behaviors of low permeability sandstones using NMR tests combined with CT analysis, and provides insights into pore structure characterization of sandstones with similar geological settings.

2. Geological settings

The Bohai Bay Basin is a large Mesozoic and Cenozoic rifted lacustrine basin located in East China (Feng et al., 2016). The Jizhong Depression is located in the west Bohai Bay Basin. The Raoyang Sag, which is an important oil- and gas-producing province, is a sub-tectonic unit lying in the southwestern part of the Jizhong Depression in the west Bohai Bay Basin (Fig. 1) (Zhang et al., 2018). Several large oil fields have been found in the Raoyang sag (Wang et al., 2008), including the giant Renqiu oil field discovered in year 1975 (Zhang et al., 2018). The Eocene and Neocene strata are divided into the Kongdian Formation (Ek), the Eocene Shahejie Formation (Es) and the Dongying Formation from bottom to top (Huang et al., 2015). The Eocene Shahejie Formation can be divided into four members (Sha-1, Sha-2, Sha-3 and Sha-4) (Zhang et al., 2016a). The Sha-3 (Es3) members (third member of Shahejie Formation) was deposited in a fan-brained delta environment in a rifted lacustrine basin (Fig. 1), in which delta plain and delta front subfacies can be divided, and the sedimentary microfacies include distributary channels, and beach bar (Fig. 1), in addition the Sha-3 member contains sandstone reservoirs and organic-rich mudstone source rocks (Zhang et al., 2018). The Lower Es3 was deposited in deep lacustrine environment, of which the lithology is dominantly dark-grey mudstones (source rocks), while the Middle and Upper Es3 mainly include fine–medium grained sandstones (main reservoir rocks) (Zhang et al., 2019).

3. Materials and theories

3.1. Experimental measurements

The NMR experiments, as non-destructive methods, were conducted firstly on the (41) plug samples of 1.5 inches in diameter and 1 inch in length. The plug samples were firstly vacuumed and then saturated with (NaCl) saline water (saturated status). NMR \( T_2 \) spectrum (incremental and cumulative \( T_2 \) distributions) of each plug sample was measured using the Oxford NMR instrument in which the frequency of magnetic field is 2 M HZ (Zhang et al., 2018). The echo interval is 0.3 ms, and the waiting time is 15,000 ms, and core samples were scanned 128 times. Then, the free water was removed by keeping the plugs in a centrifugal machine for 1 h with a constant rotation speed of 10,000 r/min, and the \( T_2 \) distributions of these samples at the centrifuged status were also measured.

Thin sections (100) were impregnated with blue-dye resin to highlight pores, and were observed under a Leica optical microscope to recognize the pore throat systems. Scanning electron microscope (SEM) analysis was performed on the carbon coated freshly broken surfaces (30 samples) to identify the pores spaces and clay minerals.

Four samples were scanned using a CT instrument under the same parameter settings at the state key laboratory of petroleum resources and prospecting. The CT scans provide 2D grayscale image at different angles for the cylindrical rock samples, and the grayscale value is strongly related to the density of the rock matrix (David et al., 2015; Liu et al., 2017). The 3D grayscale images were acquired by overlapping these 2D segmented CT images (Guo et al., 2018). The image of the cylindrical sample consists of 1449 × 1461 × 1132 elements with a voxel resolution at 1.48 μm.

The CT analysis produces two-dimensional (2D) images based on the differences in X-ray attenuation related to variations in density and composition (Kyle and Ketcham, 2015). Then the 3D images can be obtained by acquiring a set of cross-sectional 2D slices, and the 3D visualization allows three-dimensional inspection and measurement of features of interest through volume rendering technique using software Avizo (Ketcham and Carlson, 2001; Ketcham, 2005; Lai et al., 2018a). Therefore the 3D distributions of pore throat systems and fractures in sandstones are visualized (Remesyen and Swennen, 2008; Lai et al., 2017). In addition, due to the variations of CT numbers of rock matrix and pores, all the pore space can be segmented by assigning colors and opacity values using the imaging software (Ketcham, 2005), and the threshold is determined during the segmentation (Yang et al., 2016). For each pore body, the average pore radius, pore surface areas as well as pore volumes can be calculated by the rendered volumes by dispatching grayscale values to microscopic features and setting thresholds on the grayscale to segment features (Zhou et al., 2017).

3.2. Fractal models from NMR measurements

The number of pores (N(\( r \))) with pore sizes \( r \) are suggested to follow the fractal scaling law as \( N(r) \propto r^{-D} \) (Liu, 2010; Cai et al., 2010; Wang et al., 2012, 2018b; Daigle et al., 2014). According to this fractal model, the cumulative pore volume \( V_p \) in the NMR measurements can be expressed as:

\[
V_p = \frac{r_{\text{max}}^{3-D} - r_{\text{min}}^{3-D}}{1 - (r_{\text{min}}/r_{\text{max}})^{3-D}} \tag{1}
\]

Where \( r \) is the pore radius, \( D \) is the fractal dimension, and \( r_{\text{min}} \) and \( r_{\text{max}} \) are the smallest and largest pore sizes (Daigle et al., 2014; Wang et al., 2018a). Eq. (1) can be simplified as Eq. (2) since \( r_{\text{min}} \) is much smaller than \( r_{\text{max}} \) in sandstones.

\[
V_p = r_{\text{max}}^{3-D} \quad \tag{2}
\]

Additionally, Eq. (3) can be derived since the \( T_2 \) is simplified as a linear function of pore radius (Eq. (4)) as long as diffusion and bulk relaxation is ignored given a very small gradient and a short echo spacing (Coates et al., 1991; Daigle et al., 2014; Dillinger and Esteban, 2014; Sigal, 2015; Shao et al., 2017; Wang et al., 2018b):

\[
V_p = \left( \frac{T_2}{T_{\text{max}}} \right)^{1-D} \tag{3}
\]
Where $T_2$ is the transverse relaxation time, milliseconds, $V_p$ is the cumulative pore volume at $T_2$ values (Zhou and Kang, 2016; Wang et al., 2018a). $T_{2\text{max}}$ are the maximum $T_2$ values in the NMR $T_2$ spectrum (Daigle et al., 2014).

\[ \frac{1}{T_2} = \frac{1}{T_{2\text{max}}} = \rho \frac{S}{V} = \rho \frac{a}{r} \]  

(4)

where $T_{2\text{max}}$ is the surface relaxation, $\rho$ is the surface relaxivity, $S$ and $V$ are the surface areas and volume of pores, smaller pores have higher $S/V$ values but shorter relaxation time (Müller-Huber et al., 2016), and $a$ equals to 2 or 3 for the cylindrical, or spherical pores (Coates et al., 1991; Kleinberg et al., 1994; Pape and Clauser, 2009; Sigal, 2015; Müller-Huber et al., 2016; Zhang et al., 2018).

Fig. 1. The sedimentary facies map of the Sha-3 sandstones in Raoyang Sag, Bohai Bay Basin, China (Guo et al., 2013; Zhang et al., 2018; Lai et al., 2019).
Eq. (5) can be derived by applying logarithm operation of Eq. (3), therefore the fractal dimensions can be calculated from the slope (3-D) of the best fitting line by plotting \( V_p \) and the corresponding \( T_2 \) in a log-log plot (Zhang et al., 2003; Zhang and Weller, 2014; Zhou and Kang, 2016; Wang et al., 2018a).

\[
\log V_p = (3 - D) \log(T_2) + (D - 3) \log(T_{2\text{max}})
\]  

(5)

Fractal Model I

The fractal model expressed in Eq. (5) is written as Model I in this study.

However, the \( T_2 \) values do not have a linear relationship with the pore size. Conversely, the power function relation between transverse relaxation time \( T_2 \) and pore radius in low permeability sandstones can be rewritten as (Eq. (6)) (Xiao et al., 2016b; Wang et al., 2018a):

\[
T_2 = m \times r^n
\]  

(6)

Therefore, according to Eq. (6), Eq. (3) and Eq. (5) can be rewritten as:

\[
V_p = \left( \frac{T_2}{T_{2\text{max}}} \right)^{\frac{1-D}{n}}
\]  

(7)

Fractal Model II

The fractal model by taking the non-linear relationships between \( T_2 \) and pore size is written as Model II in this study (Eq. (8)). Actually, the fractal dimensions derived from Fractal Model I and Fractal Model II are linearly scaled.

Lai et al. (2018b) proposed a fractal model to calculate the fractal dimensions of the porous rocks using laboratory NMR measurements (Fractal Model III).

Suggesting the pore systems can be modeled as sphere in shape, then the numbers of pores (\( N_i \)) at a given pore size \( r_i \) (\( T_{2i} \) value) is expressed as (Eq. (9); Sigal, 2015).

\[
N_i = \frac{V_{pi}}{\frac{4}{3}} = \frac{V_{pi}}{36\pi r_i^3}
\]  

(9)

\( V_{pi} \) (%) is the pore volume measured at certain \( T_2 \) relaxation time (\( T_{2i} \), ms), \( r_i \) is the pore radius, \( \rho \) is the surface relaxivity (\( \mu \text{m}/\text{ms} \)).

Therefore the pore numbers composed of pore size larger than \( r_i \) is given by Eq. (10):

\[
\begin{align*}
\log V_p &= \frac{(1 - D)}{n} \log(T_2) + \frac{(D - 3)}{n} \log(T_{2\text{max}}) \\
&= \log V_{pi} - \frac{3}{n} \log(T_{2\text{max}})
\end{align*}
\]  

(8)

Fig. 2. Core photos showing the lithology of Shahejie Formation in Raoyang Sag, Bohai Bay Basin, China.
A. Intergranular pores with irregular shapes, **Well Liu 101**, 3696.84m

B. Intergranular (primary) pores, **Well Liu 101**

C. Partly dissolved feldspar grains, **Well Liu 101**, 3644.22m

D. Moldic pores due to complete dissolution of feldspars, **Well Liu 101**, 3696.94m

E. Feldspar remnants in the feldspar hosted dissolution pores, **Well Liu 101**, 3650.42m

F. Kaolinites within the feldspar dissolution pores, **Well Liu 101**, 3705.14m

Fig. 3. Thin section images showing the pore systems of the Shahejie sandstones in Raoyang Sag (Zhang et al., 2018).

\[
N(>r_i) = \sum_j \frac{V_{ji}}{\frac{4}{3}\pi r_j^3} \quad (10)
\]

Where \( j = i + 1 \).

By combining Eq. (9), and Eq. (10), Eq. (11) can be derived:

\[
N(>r_i) = \sum_j \frac{V_{ji}}{36\pi (\rho T_d)^3} = \sum_j \frac{V_{pi}}{36\pi\rho^3(T_d)^3} \propto (3\rho T_d) - Df \quad (11)
\]

Eq. (12) could be derived by taking the logarithm on the both sides of the equations.
A. Feldspar dissolution pores, Well Liugu 2

B. Honey-comb like illite/smectite mixed layers are micro-porous, Well Liu 101

C. Micro-porous chlorites, Well Liu 101

D. Micro-porous illites and illite/smectite mixed layers, Well Liu 101

E. Micro-porous illite/smectite mixed layers, Well Liu 101

F. Micro-porous illite/smectite mixed layers, Well Liu 101

Fig. 4. SEM images showing the clay minerals and micropores (Zhang et al., 2016a, 2016b, 2018).

\[
\log \sum_{j} \frac{V_{pj}}{(T_{2j})^{3}} + \log A = -Df \log(T_{2})
\]

(12)

Fractal Model III Where \( A = \frac{1}{3(p+1)} \), and \( B = 3p \). Therefore the fractal dimension \( (Df) \) can be derived according to the relationships between \( \sum_{j} \frac{V_{pj}}{(T_{2j})^{3}} (N(r_{j})) \) and \( T_{2j} \).

The fractal model proposed by Lai et al. (2018b) is written as Fractal Model III.

4. Results

4.1. Lithology and pore systems

The Raoyang Sag, a small but petrolierous sag, is a subectonic unit...
lying in the southwest of the Jizhong depression, Bohai Bay Basin, East China (Zhang et al., 2018). The Eocene Shahejie Formation suggested to be deposited in the fan-braided delta environment facies in a rifted lacustrine basin (Fig. 1) (Zhang et al., 2019). The lithology of the Eocene Shahejie Formation in Raoyang Sag includes fine-medium grained sandstones, siltstones, dark-grey mudstones and conglomerates, and abundant beddings (parallel, wavy, ripples) can be observed (Fig. 2A-D) (Cao et al., 2013; Zhang et al., 2016a, 2018).

Microscopic observations (thin section) and SEM images confirm that pore systems of Sha-3 sandstones contain various pore types (intragranular, intergranular, microfracture, molds, micropores) which span a wide ranges of pore sizes. The primary residual intergranular pores, which have irregular shapes, are occasionally observed, and they are mainly associated with the fine-medium grained sandstones with well sorting (Fig. 3A-B). Conversely, the intragranular dissolution pores or even moldic pores, which are derived from partly to complete dissolution of framework grains (dominantly feldspars), are abundant in the Sha-3 sandstones (Fig. 3C-D). It is worth mentioning that the secondary dissolution pores commonly coexist with the primary intergranular ones (Fig. 3A-D) (Zhang et al., 2016a). There are some remnants remaining in the feldspar hosted dissolution pores (Fig. 3E), and additionally, some kaolinites are associated with the feldspar dissolution pores (Fig. 3F) (Zhang et al., 2019).

Abundant micropores (< 10 μm), which are below the resolution of the microscopic thin section analysis, are observed in the Sha-3 sandstones under the SEM observations. There are some honeycomb-like
dissolution pores in the feldspar grains (actually feldspar grains commonly contain remnant due to partly dissolution) (Fig. 4A). Additionally, there are abundant intercrystalline micropores associated with authigenic clay minerals (dominantly chlorite, illite and mixed layered illite/smectite) (Zhang et al., 2016b)(Fig. 4B–D) (Zhang et al., 2019). Especially the illite and mixed layered illite/smectite, which are commonly observed in the Sha-3 sandstones and are mainly associated with the diagenetic feldspars, contain abundant intercrystalline micropores (Fig. 4E and F) (Zhang et al., 2018, 2019).

4.2. CT scanning images

According to the fact that the grey value is density-based, the higher the density of rock matrix, the higher the grey values are and the brighter the voxels will be (Liu et al., 2017; Zhao et al., 2018). Therefore, the white areas in Fig. 5A refer to the carbonate (calcite and dolomite) cements due to its higher density as revealed by the previous studies (Zhang et al., 2016a,b; Zhang et al., 2018). The grey areas in the 2D images are the framework grains, and the thin section petrography indicates that quartz and feldspars are the predominant rock composition (Fig. 5E). In the 2D slices the pore spaces correspond to the blue areas (Fig. 5A). By the step of overlapping the 2D segmented CT images, the pore networks can be extracted (Fig. 5B), and the narrow space is defined as throats, while greater space is defined as pores when extracting the pore network models (Yang et al., 2016).

The reconstructed 3D image is shown in Fig. 5C, in which the pore clusters are evident. It can be concluded that the matrix composition is composed of quartz, feldspar and cements. The intergranular pores, which can be observed in the thin section petrography (Fig. 5E), showed some connectivity with adjacent isolated micropores (Fig. 5C). Isolated pore clusters with complex shapes are also commonly identified by CT images. Quantitative CT analysis reveals that the pore diameter of the four samples ranges from 1.48 μm to about 131.906 μm. The resolution value is 1.48 μm, and it’s worth mentioning that there are small pores (Fig. 5F) below the resolution of CT analysis, but can be detected by the SEM analysis, and abundant micropores are related to the illite and mixed layer illite/smectite (Fig. 5F) (Zhang et al., 2018). The pores are assumed to be equivalent spheres when measured, however, the actual pore radius might be quite different from the equivalent spherical radius (Guo et al., 2015).

4.3. NMR T2 spectra

Theoretically, the small-scale pore networks below the CT scanning (< 1.48 μm) can also be characterized in NMR analysis by shortening the echo interval during measurements (Gao and Li, 2016). An increased resolution in CT scanning implies a reduction of the sample size (Lai et al., 2017; Liu et al., 2017). Unlike the CT scanning, the NMR T2 spectrum almost gives the full range of pore body size distribution (Xiao et al., 2016a).

Typical T2 includes the incremental and cumulative curve measured under saturated and centrifuged conditions (Fig. 5D) (Zhang et al., 2018). The bimodal T2 behavior spectrum (left-skewed: high left peak but low right peak) (Fig. 5D) suggests the presence of two populations of pore sizes, and this is in accordance with the coexistence of large-scale pores revealed in the thin section petrography and the small-scale pores detected in the SEM image (Fig. 5E-F). The dominant pore size distribution has a wide range from 0.1 to 100 ms (Fig. 5D). The T2cutoff divides the T2 spectrum into two parts: (1) free fluid index (FFI) present in large-scale pore, and (2) bulk volume of immovable fluids (BVI) which are associated with micropores and clay minerals (Dillinger and Esteban, 2014). Gao and Li (2015) documented the procedure to determine the T2cutoff values from the T2 spectrum, and the T2cutoff in Fig. 5D is determined as 3.2 ms. High content of BVI (average: 65.4%) is

![Fig. 6. The double-logarithm coordination showing the relationship between the cumulative pore volume and the transverse relaxation time T2 (Fractal Model I).](image1)

![Fig. 7. Pore radius and the pore volume at each pore radius from CT scanning.](image2)
5. Discussion

5.1. Fractal behaviors of NMR T2 spectrum

The fractal curves are the representation of the relationships between the number of pores (> r) and the pore throat radius (r) (Wang et al., 2018). The T2cutoff is an important parameter linking microscopic geometries with their macroscopic performance (Lai et al., 2018a), and it has a wide range from 1.04 to 43.29 ms with an average of 12.46 ms.

5.1.1. Fractal model I

From Eq. (5) (Fractal Model I), it can be concluded that the relationship between Vp and T2 will be linear on a log–log plot (Wang et al., 2018b). However, by plotting logVp against logT2 for all the samples, not a straight line but instead a curve and a straight line can be observed (Fig. 6). The fractal curves break into two segments at the T2cutoff value (Shao et al., 2017) (Fig. 6), and this phenomenon is similar to the fractal behavior of Donghetang sandstones which have moderate reservoir quality (Wang et al., 2018b). For the large-scale pore networks, which are associated with the T2 ranges larger than T2cutoff, high coefficient of determination is observed between logVp and logT2, and this implies that the pore systems consisting of large pore throats are fractal according to the Fractal Model I. The fractal dimension is determined as 2.8399 according to Eq.(5) and logVp and logT2 in Fig. 6. The fractal dimension approaching 3.0 indicates a complex pore size distribution in the large-scale range. However, for the small-scale pore networks (T2 < T2cutoff), the logVp-logT2 plot is not a straight line but instead a smooth curve, and the coefficients of linear regressions are
much lower than that of large-scale pores (Fig. 6). In addition, the small pores will derive an apparent fractal dimension values less than 2.0 since the slope of the best fit line is larger than 1.0 (Fig. 6).

5.1.2. Fractal model

Sandstones (especially the low permeability sandstones or tight sandstones) are commonly characterized by a large range of pore sizes ranging from the nano-scale to micro-scale, and have complex pore geometry (Pape and Clauser, 2009; Lai et al., 2018a). Therefore the surface relaxivity $\rho$, which is used to convert NMR relaxation time distributions to pore size distributions (Daigle et al., 2014), is not a constant, but may decrease with pore radius (Pape and Clauser, 2009). In addition, the pores were assumed to be spherical and that the magnetic field gradient was constant and equal in all pores, and that the entire pore system has a uniform surface relaxivity in the fractal model (Eq. (1)-Eq. (5)) (Daigle et al., 2014; Daigle and Johnson, 2016).

However, the natural sandstones exhibit much more complex pore system geometry (Daigle and Johnson, 2016). The CT images show that the pores with irregular morphology are scattered in the 3D space (Fig. 5). The NMR $T_2$ distribution is expressed by the pore radius distribution curve where each pore radius class is represented by its volume (Pape and Clauser, 2009). Also, the CT scanning can provide the data about the pore radius and the pore volume at each pore radius (Fig. 7). The pore size distribution from quantitative CT analysis at the pore radius ranging from 9.18 to 53.78 $\mu$m is characterized by a multimodal behavior, which is not as smooth as the NMR $T_2$ spectrum (Fig. 7).

Therefore, the real pore networks tend to have a much higher surface area to volume ratio than cylinders or spheres (Daigle and Johnson, 2016). In addition, the fluids in smaller pores experiences stronger surface relaxation than in large pores due to higher surface-to-volume ratio of small pores in the NMR $T_2$ spectrum (Liu et al., 2017). Consequently, in the low permeability sandstones, especially in the small-scale pore networks ($T_2 < T_{2\text{cutoff}}$), the conditions under which $T_2$ is directly proportional to $r$ are often not met, and this is mainly attributed to the additional relaxation contribution from diffusion in internal field gradients (Daigle et al., 2014). In such cases, the $T_2$ values do not scale linearly with pore size, and the fractal dimensions can't be determined from the logVp-log$T_2$ plot (Daigle et al., 2014). Strong power correlation relationship ($R^2 > 0.81$) but not linear relationship is observed between the surface area to volume ratio (S/V) and the pore radius ($r$) (Fig. 8), therefore the pore radius will not be linearly scaled with the NMR $T_2$ values, which is proportional to the surface area to volume ratio (Fig. 8). Therefore Eq. (8) should be used to derive the fractal dimensions according to the power function relation between transverse relaxation time $T_2$ and pore size (Fractal Model II) (Xiao...
in the CT scanning, actually, the smallest pore size in Fig. 8 is 9.18 μm, which means the smallest pores can be described as both pore throats and pores (Luo et al., 2017). Fractal analysis shows that the fractal dimensions approaching 3.0, which are calculated by the fractal model I and fractal model II, are too high for low permeability sandstones. In addition, the CT scanning data (Fig. 5) also prove that this sample pore connectivity has relatively poor pore connectivity since most of the large pore bodies are not connected by effective pore throats (Fig. 5A–C).

In Fig. 10, the pore space appears in blue and the matrix (quartz, feldspar, and carbonate cements) is transparent (Fig. 10A–C). The NMR T2 spectrum also shows a bimodal distribution behavior, which is left-skewed (Fig. 10D), and this indicates a geometrical arrangement composed of small to large pore size domains (Lai et al., 2018b). The dominant T2 distribution appears as a major dominant peak (T2peak) at the shorter T2 times (about 1 ms), which is different from normal sands (Rezaee et al., 2012).

Similarly, the logVp-logT2 plot (fractal model I) is also not a straight line, but instead breaks into two segments, however, the inflection point is at the position where T2 < T2cutoff (Fig. 10E), some of the logVp-logT2 plots break into two segments at T2 < T2cutoff, especially the samples with high permeability and high T2cutoff values (Fig. 10F; Fig. 6). The linear trendline with a regression coefficient R² > 0.95 supports the large pore realm corresponding to long T2 components is in fact fractal (Fig. 10E). Small-scale pores (short T2 components) tend to have a slope of 2.5981 through linear regression, and this will result in a calculated fractal dimension value larger than 3.0, while large-scale pores (long T2 components) have a slope of 0.1703 by linear regression, and gives a fractal dimension of 2.8297 (Fig. 10E).

Another example is presented in Fig. 11, and the fractal model I and Fractal Model III were used to derive the fractal dimensions, as can be seen, the fractal model I has an inflection point which means that not all the pore systems can be described by the fractal model, and only the large-scale pore systems (T2 > 1.14 ms) can be characterized. However, when using the fractal model III, almost all the pore systems can be characterized (0.03–613.7 ms) (Fig. 11).

Therefore, from the aspect of NMR measurements, the small pore and pore throats with small size (short T2 components) do not follow the same fractal law with the large-scale pore networks if using the fractal model I. The reasons should be that the small pore realm with short T2 components, including micropores (clay mineral dominated) and the pore throats, account for a large proportion of the total pore systems (Lai et al., 2018a), but they are not self-similar to the large-scale pore systems (Wang et al., 2018b).

The inflection points in the fractal model I and fractal model II are hard to be determined, making the determination of fractal dimensions difficult and could be varied by individuals (Fig. 6; Figs. 10 and 11). Conversely, when using the Fractal Model III, the human factors could be avoided since almost all the entire pore size distributions could be described by the fractal model, and no inflection points exist. Therefore, the fractal model III have advantages in fractal characterization of pore structures in low permeability sandstones (Figs. 10 and 11).

5.2. Fractal dimensions

In low permeability or tight sandstones, the micropores (< 10 μm) act as both pore throats and pores (Luo et al., 2017). Fractal analysis reveals that the pore bodies with large pore radius don’t exhibit similar fractal scaling with the small-scale pore throat systems (Daigle et al., 2016b; Wang et al., 2018a):

Consequently, the fractal dimension can be derived from the slope of the best fitting line of the logVp-logT2 plot providing parameter n is known. The value of “n” according to CT analysis for the four samples is determined as 0.5225 (Fig. 8). Then the fractal dimension of the samples in Fig. 6 is determined as 2.931 (3–0.5225 × 0.1335), indicating a very complex pore throat structure in the Sha-3 sandstones.

The fractal dimensions calculated by Fractal Model II is linear with the fractal dimensions calculated by fractal model I.

5.1.3. Fractal model III

Fig. 8 shows the power relationship between S/V and pore size. However, as previously clarified, the resolution of the CT scanning in this study is 1.48 μm, which means the smallest pores can be described in the CT scanning, actually, the smallest pore size in Fig. 8 is 9.18 μm (Fig. 8). The complex deep-burial diagenetic modifications (compaction, quartz, carbonate and clay mineral cementation and framework grain dissolution) change the amount and distribution of pore spaces, creating smaller and more disconnected (more and more sphere) pores (Higgs et al., 2007; Cook et al., 2011; Xi et al., 2015; Lai et al., 2018b). Therefore, the small pore sizes should also be taken into consideration in the fractal models.

Presented in Fig. 9 is the plot of log(N(rj)) versus log(T2) (the same sample in Fig. 6), and it can be observed that there is no evident inflection point existing, which implies that almost all the entire pore systems (the T2 values ranging from 0.05 to 174 ms) can be described by the fractal model III (Fig. 9). Additionally, the slope of the double-logarithm coordination indicates a fractal dimension of 2.871, and it is similar to the fractal dimension calculated by the fractal model I and Fractal Model II (Fig. 6). Generally speaking, the fractal dimensions have advantages in fractal characterization of pore structures in low permeability sandstones (Figs. 10 and 11).
Considering the effect of internal magnetic field gradients, Daigle et al. (2014) noted that fractal dimension can be derived from the T2 distribution in rocks with smaller pores, since the surface relaxation is dominated in the overall T2 values, while in rocks with larger pores the internal gradient effect is most significant.

The log Vp is plotted against log T2 for all the 41 samples used for NMR measurements, and the fractal dimensions of the NMR T2 spectra are calculated using the fractal model I and fractal model II. The high fractal dimensions (D2) (> 2.7) indicate that the large-scale pore network turns out to be complex, and conclusions can be made that both the NMR parameter (T2gm) and reservoir property (permeability) have negative correlations with fractal dimensions, with coefficient of determination of 0.81 and 0.64, respectively (Fig. 12). Increasing fractal dimension implies the transformation of the pore shapes from regular to rough. The roughness of the pore surface, and irregular and complex pore size distribution hinder the migration of pore fluid (Shao et al., 2017), and therefore contribute to a low T2gm value as well as permeability (Fig. 12).

The BVI values from NMR measurements are high, and have a wide range from 31.49% to 97.93% (Zhang et al., 2018). The NMR T2 distribution larger than T2cutoff, which mainly corresponds to the large intergranular pores connected by effective pore throats (Zou et al., 2012), can be described by the fractal model I (Fig. 10E). The large pore realm with long T2 components (> T2cutoff), which are treated effectively movable in the NMR distribution, are connected by pore throats, and can make a significant contribution to the macroscopic performances (permeability) (Lai et al., 2016). Therefore, though the complexity and heterogeneity of the pore structure are dependent on the coexistence of large-scale pores and micropores, and the micropores may account for a large portion of the total porosity, the macroscopic reservoir quality of low permeability sandstones could be determined by the distribution of the large-scale pore systems (Lai et al., 2018a).

5.3. Fractal characterization and pore networks

As is well-known, the fractal dimension increases continuously as the complicacy of pore increases (Wang et al., 2012), therefore, the fractal dimension approaching 3.0 implies a heterogeneous pore structure. The calculated moderate to high values of fractal dimensions calculated by fractal model III (D3) (ranging from 2.600 to 2.985, and averaging as 2.826) indicate the medium to high complexity and heterogeneity of the pore structures, and this is in consistent with the CT
scanning and thin section data. Additionally, both the $T_{2\text{gm}}$ and permeability are strongly negatively correlated with fractal dimensions calculated by fractal model III, with coefficients of determination of 0.916 and 0.70, respectively (Fig. 13).

Furthermore, the fractal dimensions calculated by fractal model III (D3) and fractal dimensions calculated by fractal model I (D1) are correlated with the NMR BVI values, respectively, and it is concluded that the D3 have stronger ($R^2 > 0.73$) correlation relationships with BVI compared with D1 ($R^2 = 0.51$) (Fig. 14). The fractal dimension D3 can determine the content of irreducible water in the rocks, and the higher the fractal dimension, the higher the irreducible water content (Fig. 14).

The fractal model proposed by Lai et al. (2018b) (fractal model III) could be used to calculate the fractal dimensions of the almost the entire pore size distribution in low permeability sandstones with laboratory NMR measurements, and the calculated fractal dimension can evaluate the heterogeneity of the porous rock satisfactorily. The fractal model III has advantages in fractal characterization of pore structures in low permeability sandstones compared with fractal model I and fractal model II.

6. Conclusions

The NMR $T_2$ spectra of the low permeability sandstones show either uni- or bimodal behaviors. Fractal analysis shows that only the large-scale pore networks can be described by the fractal geometry when using the fractal model I and fractal model II, and the fractal curves ($\log V_p - \log T_2$) break into two segments at the $T_{2\text{cutoff}}$ value or smaller, and it is hard to determine the inflection points. Quantitative CT analysis reveals that pore surface area to volume ratio ($S/V$) shows a power relationship with pore radius, and therefore the pore radius is also a power function of $T_2$. The oversimplification of pore shapes, variation of magnetic field gradient and higher surface-to-volume ratio of small pores, results in the deviations of the fractal dimensions of small pore realms. The entire pore size distributions can be described by the fractal model III, and the calculated fractal dimensions are in accordance with the CT scanning and thin section data, and are strongly correlated with the $T_{2\text{gm}}$ and permeability as well as BVI values.

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Appendix A. Supplementary data

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References


